Model Questions

B.Sc. Sem - II (Paper - Mat C3)

Analysis - I

Short answer type Question.

1). a) State the order-Completeness of the set of Real nos.
b) Define Neighbourhood of a point.
c) State Bolzano - weirstrass theorem for set
d) Define a monotonic sequence.

2). a) Define Absolute and Conditional convergence.
b) Define an Alternating Series.
c) State Cauchy's General Principle of Convergence for series.
d) State Pringsheim's Theorem.

3). Examine the convergence
a) $\sum \sqrt{\frac{n}{n^4+2}}$

b) $\sum \left(\sqrt{n^4+1} - \sqrt{n^4-1}\right)$

4) Examine the convergence
a) $\sum n^2$

b) $\sum \left(\frac{1}{n}\right)^{1/n}$

c) $\sqrt{\frac{1}{4}} + \sqrt{\frac{2}{6}} + \cdots + \sqrt{\frac{n}{2(n+1)}} + \cdots$

5) Prove that limit of a sequence if it exist is unique.

6). Prove that every Convergent sequence is bounded but the converse is not true.

7) Prove that a sequence converges iff it is a cauchy sequence.

8). If $\lim f_n = K$ and $\lim \phi_n = l$ then Prove that

i) $\lim (f_n + \phi_n) = K + l = \lim f_n + \lim \phi_n$

ii) $\lim (f_n \cdot \phi_n) = Kl = (\lim f_n) \cdot (\lim f_n)$.

9) Examine the convergence of the following series.

a) $\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \cdots$

b) $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots$ for +ve values of x

10). State and Prove Cauchy's nth root test

Long answer type question.

1). State and Prove D'Alembert's Ratio Test.

2) State and Prove Raabe's Test.

3) Discuss the convergence of the series
$x^2(\log 2)^p + x^3(\log 3)^p + \dots + x^n(\log n)^p + \dots$

4) State and Prove Cauchy's Integral Test.

5) Prove that every absolutely convergent series is convergent.

6). The necessary and sufficient condition for a real no 's' to be the supremum of a bounded above set S is that 's' must satisfy the following conditions
a) $x \le s \ \forall \ x \in S$
b) for each +ve real no. ε, $\exists$ a real no. $x \in S$ such that $x > s - t$.

7). P.T. a non-empty subset of real nos. which is bounded below has the greatest lower bound in R.

8). P.T. every monotonically increasing sequence which is bounded above converges to its l.u.b.

9). State and Prove Monotone convergence Theorem.

10) P.T. intersection of a finite no. of open sets is an open set